Unit 15

Reduction of State Tables / State Assignment

FSM Design Flow

Statement of Problem

⇩

State Graph

⇩

State Table

⇩

Reduction of States

⇩

State Assignment

⇩

Choice of F/F

⇩

Derivation of F/F Input Equation and Z Output Equation

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Circuit Realization and Timing Chart

Outline

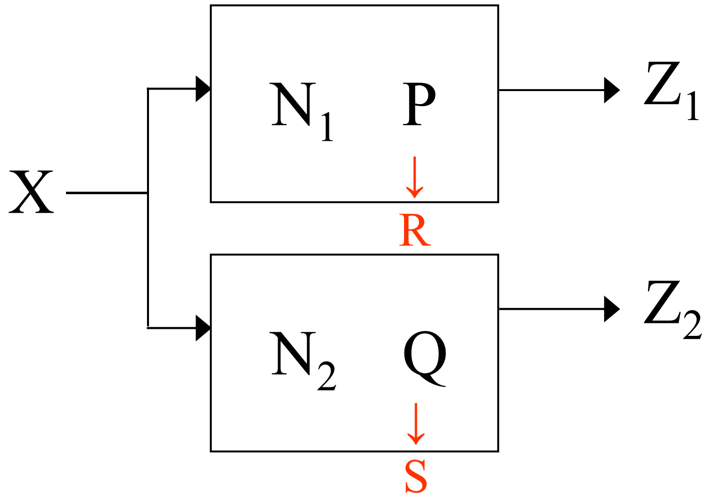
- Equivalent states
- Elimination of redundant states
- Determination of state equivalence using an implication table
- Equivalent sequential circuits
- Incompletely specified state tables
- Derivation of F/F input equations
- Equivalent state assignments
- Guidelines for state assignment
- Using a One-Hot state assignment

Equivalent States

Equivalent states "≡"

2 machines: N_1, N_2

- state P in N_1
- state Q in N_2



$P \equiv Q$ iff $R \equiv S$ and $Z_1 = Z_2$

$\lambda(P,x) = \lambda(Q, x)$: output

$\delta(P,x) = \delta(Q, x)$: next state

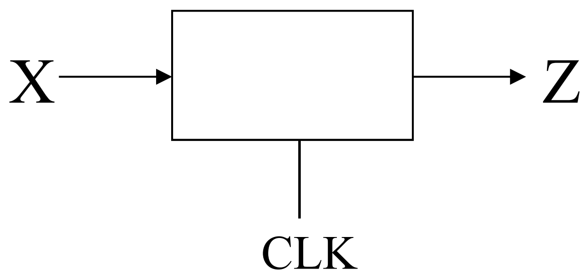
x: any single input

Elimination of Redundant States (1/5)

When setting up states, some extra states may be included

⇨ eliminate these states

A previous example



Check 4 consecutive inputs as a group , then reset

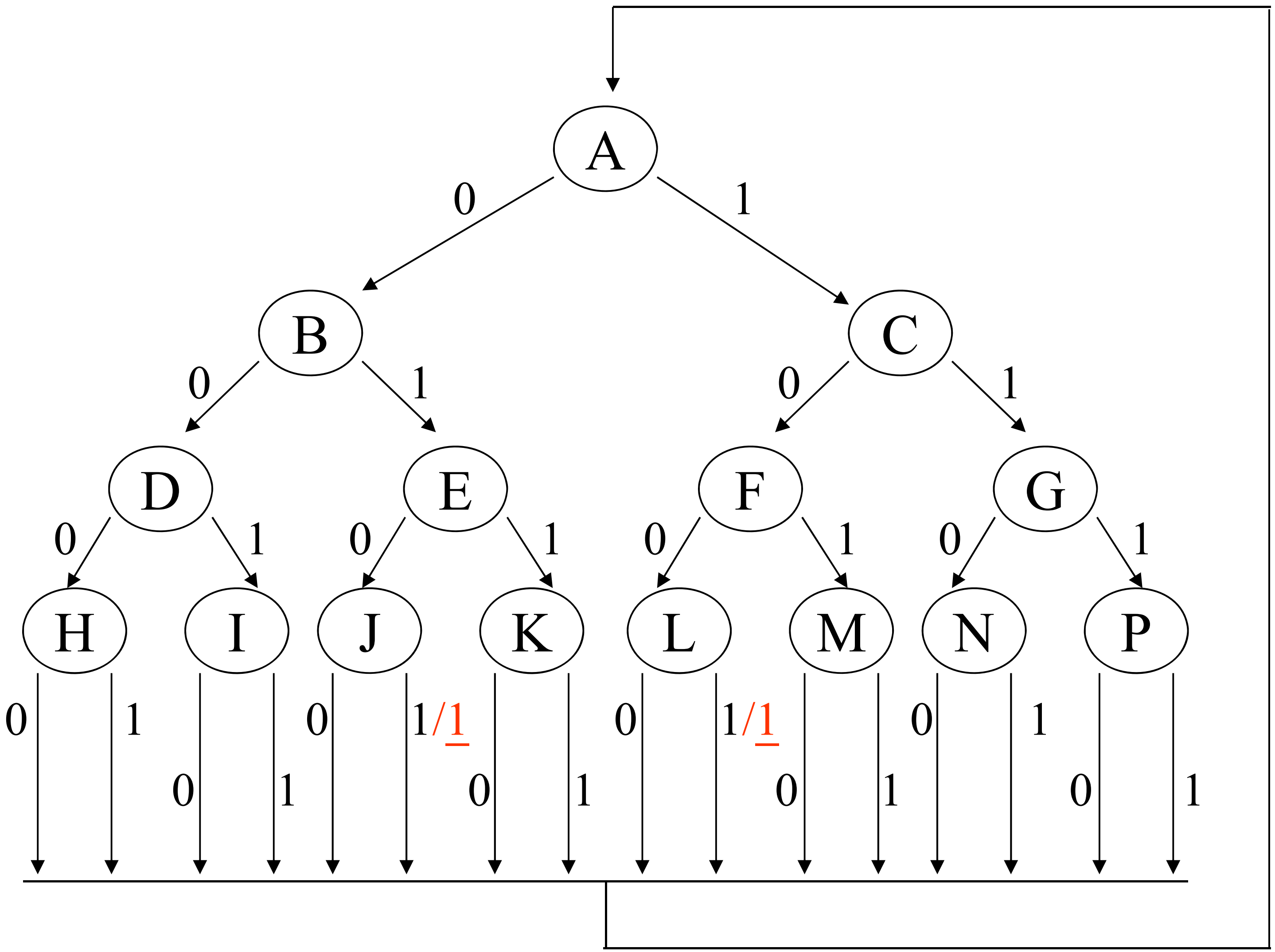
Z = 1 when X = 0101 or 1001

A brute force approach:

Reset state A, checks 3 consequent bits of <u>every possible combinations</u>.

After the 4th bit coming-in, give output and reset to state A

Elimination of Redundant States (2/5)



Elimination of Redundant States (3/5)

Input Sequence	P.S.	N.S. x = 0	N.S. x = 1	Present Output x = 0	Present Output x = 1
Reset	A	B	C	0	0
0	B	D	E	0	0
1	C	F	G	0	0
00	D	H	I	0	0
01	E	J	K	0	0
10	F	L	M	0	0
11	G	N	P	0	0
000	H	A	A	0	0
001	I	A	A	0	0
010	J	A	A	0	1
011	K	A	A	0	0
100	L	A	A	0	1
101	M	A	A	0	0
110	N	A	A	0	0
111	P	A	A	0	0

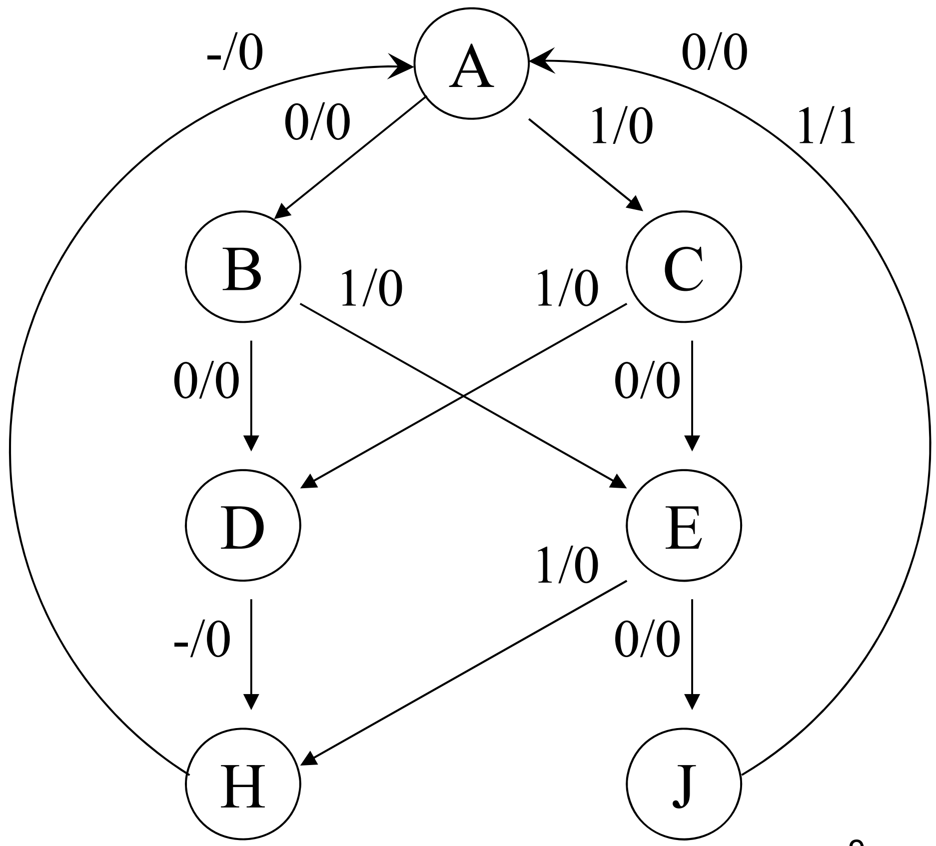
Elimination of Redundant States (4/5)

Equivalent states: the same N.S. & output

Input Sequence	P.S.	N.S. x = 0	N.S. x = 1	Present Output x = 0	Present Output x = 1
Reset	A	B	C	0	0
0	B	D	E	0	0
1	C	E ~~F~~	D ~~G~~	0	0
00	D	H	H ~~I~~	0	0
01	E	J	H ~~K~~	0	0
→ 10	E ~~F~~	J ~~L~~	H ~~M~~	0	0
→ 11	D ~~G~~	H ~~N~~	H ~~P~~	0	0
000	H	A	A	0	0
→ 001	H ~~I~~	A	A	0	0
010	J	A	A	0	1
→ 011	H ~~K~~	A	A	0	0
→ 100	J ~~L~~	A	A	0	1
→ 101	H ~~M~~	A	A	0	0
→ 110	H ~~N~~	A	A	0	0
→ 111	H ~~P~~	A	A	0	0

Elimination of Redundant States (5/5)

P.S.	N.S. x = 0	N.S. x = 1	Output x = 0	Output x = 1
A	B	C	0	0
B	D	E	0	0
C	E	D	0	0
D	H	H	0	0
E	J	H	0	0
H	A	A	0	0
J	A	A	0	1



Determination of State Equivalence by Using an Implication Table (1/4)

Ex:

P.S.	N.S. x = 0	N.S. x = 1	Z
a	~~d~~ a	c	0
b	f	h	0
c	~~e~~ c	~~d~~ a	1
~~d~~	~~a~~	~~e~~	~~0~~
~~e~~	~~c~~	~~a~~	~~1~~
f	f	b	1
g	b	h	0
h	c	g	1

Determination of State Equivalence by Using an Implication Table (2/4)

Implication chart construction

1. State comparison by implied pair (only for the-same-output pair)

Output shall be the same

	a	b	c	d	e	f	g
b	d – f c – h						
c	X	X					
d	a – d c – e	a – f e – h	X				
e	X	X	a – d c – e	X			
f	X	X	b – d e – f	X	a – b c – f		
g	b – d c – h	b – f	X	a – b e – h	X	X	
h	X	X	d – g c – e	X	a – g	b – g c – f	X

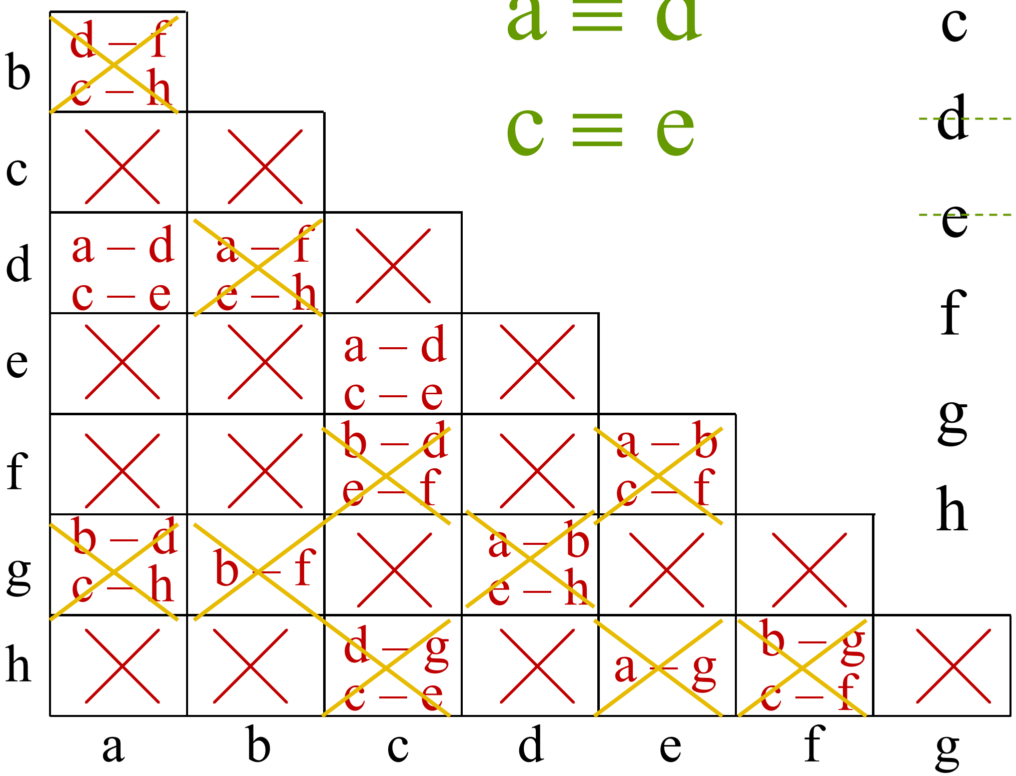
P.S.	N.S. x = 0	N.S. x = 1	Z
a	d	c	0
b	f	h	0
c	e	d	1
d	a	e	0
e	c	a	1
f	f	b	1
g	b	h	0
h	c	g	1

Determination of State Equivalence by Using an Implication Table (3/4)

2. Check implied pair iteratively

$a \equiv d$

$c \equiv e$



P.S.	N.S. x = 0	N.S. x = 1	Z
a	~~d~~ a	c	0
b	f	h	0
c	~~e~~ c	~~d~~ a	1
~~d~~	~~a~~	~~e~~	~~0~~
~~e~~	~~c~~	~~a~~	~~1~~
f	f	b	1
g	b	h	0
h	c	g	1

Determination of State Equivalence by Using an Implication Table (4/4)

$a \equiv d$

$c \equiv e$

	N.S.		
P.S.	x = 0	x = 1	Z
a	a	c	0
b	f	h	0
c	c	a	1
f	f	b	1
g	b	h	0
h	c	g	1

Equivalent Sequential Circuits (1/3)

Definition : $N_1 \equiv N_2$ if P in $N_1 \equiv$ Q in N_2
or S in $N_2 \equiv$ T in N_1

Ex:

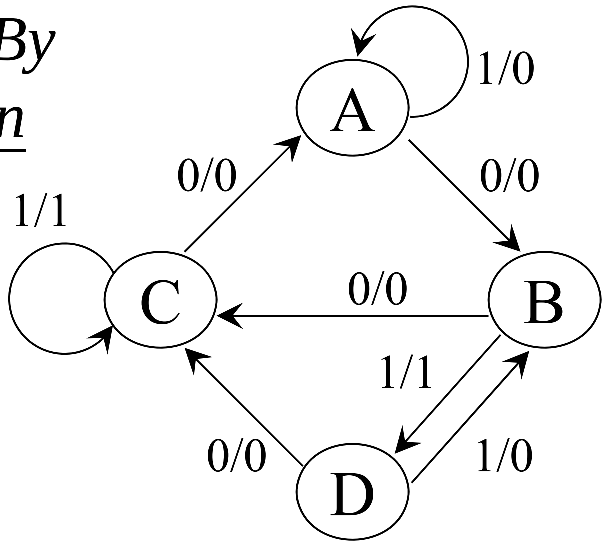
N_1

	$x=0$	$x=1$	$x=0$	$x=1$
A	B	A	0	0
B	C	D	0	1
C	A	C	0	1
D	C	B	0	0

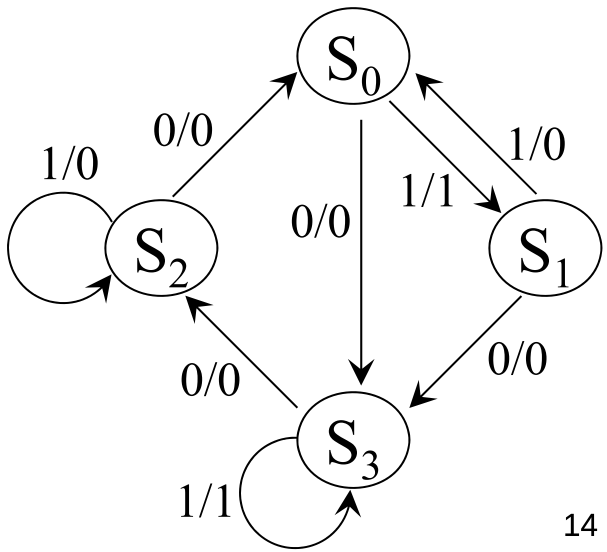
N_2

	$x=0$	$x=1$	$x=0$	$x=1$
S_0	S_3	S_1	0	1
S_1	S_3	S_0	0	0
S_2	S_0	S_2	0	0
S_3	S_2	S_3	0	1

Method 1:By *<u>observation</u>*



$A \equiv S_2$
$B \equiv S_0$
$C \equiv S_3$
$D \equiv S_1$



Equivalent Sequential Circuits (2/3)

N_1

	$x=0$	$x=1$	$x=0$	$x=1$
A	B	A	0	0
B	C	D	0	1
C	A	C	0	1
D	C	B	0	0

N_2

	$x=0$	$x=1$	$x=0$	$x=1$
S_0	S_3	S_1	0	1
S_1	S_3	S_0	0	0
S_2	S_0	S_2	0	0
S_3	S_2	S_3	0	1

Method 2: By implication table

1. Construct implication table by listing all state pairs. X out pairs with different outputs

	A	B	C	D
S_0	X	$C-S_3$ $D-S_1$	$A-S_3$ $C-S_1$	X
S_1	$B-S_3$ $A-S_0$	X	X	$C-S_3$ $B-S_0$
S_2	$B-S_0$ $A-S_2$	X	X	$C-S_0$ $B-S_2$
S_3	X	$C-S_2$ $D-S_3$	$A-S_2$ $C-S_3$	X

Equivalent Sequential Circuits (3/3)

	A	B	C	D
S_0	X	$C - S_3$ $D - S_1$	$A - S_3$ $C - S_1$	X
S_1	$B - S_3$ $A - S_0$	X	X	$C - S_3$ $B - S_0$
S_2	$B - S_0$ $A - S_2$	X	X	$C - S_0$ $B - S_2$
S_3	X	$C - S_2$ $D - S_3$	$A - S_2$ $C - S_3$	X

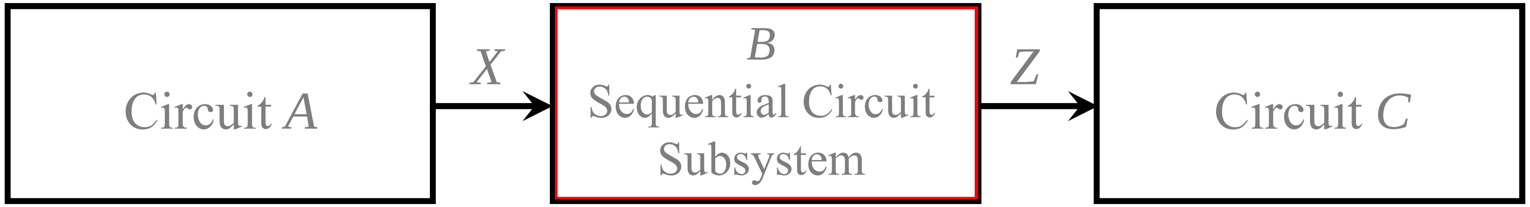
2. X out additional nonequivalent state pairs

	A	B	C	D
S_0	X	$C - S_3$ $D - S_1$	~~$A - S_3$~~ ~~$C - S_1$~~	X
S_1	~~$B - S_3$~~ ~~$A - S_0$~~	X	X	$C - S_3$ $B - S_0$
S_2	$B - S_0$ $A - S_2$	X	X	~~$C - S_0$~~ ~~$B - S_2$~~
S_3	X	~~$C - S_2$~~ ~~$D - S_3$~~	$A - S_2$ $C - S_3$	X

$\Rightarrow A \equiv S_2 \quad B \equiv S_0 \quad C \equiv S_3 \quad D \equiv S_1$

Incompletely Specified State Tables (1/3)

Ex:



A can only give 100, 110

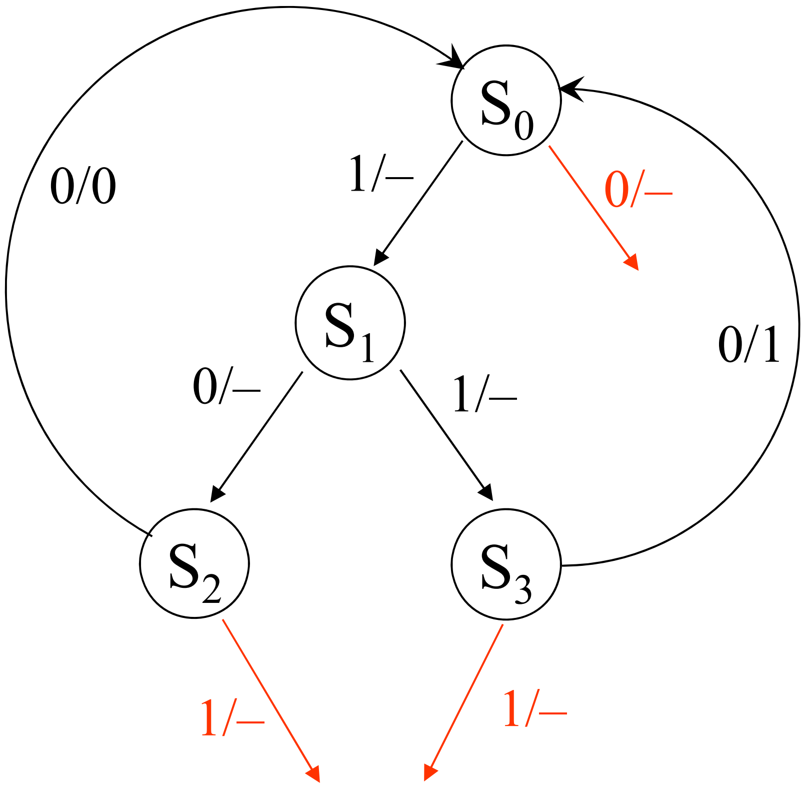
$$\text{B gives} \begin{cases} \text{“0”} & \text{if} \quad 100 \ \text{received} \\ \text{“1”} & \quad\;\;\, 110 \ \text{received} \end{cases} \text{at the } 3^{rd} \text{ bit}$$

	t_0	t_1	t_2		t_0	t_1	t_2
X =	1	0	0	Z =	–	–	0
	1	1	0		–	–	1

If don't cares are present, the state table is called incompletely specified

Incompletely Specified State Tables (2/3)

State Graph



State Table

P.S.	N.S. x=0	N.S. x=1	Z x=0	Z x=1
S_0	–	S_1	–	–
S_1	S_2	S_3	–	–
S_2	S_0	–	***0***	–
S_3	S_0	–	***1***	–

Incompletely Specified State Tables (3/3)

P.S.	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
S_0	– (S_0)	S_1	– (0)	–
S_1	~~S_2~~ S_0	S_3	– (1)	–
S_2	S_0	– (S_1)	***0***	–
S_3	S_0	– (S_3)	***1***	–

⇩ States reduction

P.S.	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
S_0	S_0	S_1	***0***	–
S_1	S_0	S_1	***1***	–

Derivation of F/F Input Equations (1/22)

State Graph

⇨ State Table

⇨ State Assignment

⇨ State Transition Table

Choice of F/F!

⇨ K-Map

⇨ F/F Input Equations

Derivation of F/F Input Equations (2/22)

Type of F/F	Input	Q=0		Q=1	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1
D F/F	D	0	1	0	1

Derivation of F/F Input Equations (3/22)

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	×

Q+

D F/F Input Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	×

$$D = Q'A'B' + QB' + AB'$$

Derivation of F/F Input Equations (4/22)

Type of F/F	Input	Q=0		Q=1	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1
D F/F	D	0	1	0	1
T F/F	T	0	1	1	0

Derivation of F/F Input Equations (5/22)

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	×

Q+

T F/F Input Map

AB \ Q	0	1
00	0	0
01	1	1
11	0	1
10	1	×

$T = A'B + AB' + QB$

Derivation of F/F Input Equations (6/22)

Type of F/F	Input	Q=0		Q=1	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1
D F/F	D	0	1	0	1
T F/F	T	0	1	1	0
S-R F/F	S	0	1	0	x
	R	x	0	1	0

Derivation of F/F Input Equations (7/22)

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	×

Q+

S-R F/F Input Map

AB \ Q	0	1
00	0	×
01	1	0
11	0	0
10	1	×

$S = AB' + Q'A'B$

AB \ Q	0	1
00	×	0
01	0	1
11	×	1
10	0	×

$R = QB$

Derivation of F/F Input Equations (8/22)

Type of F/F	Input	Q=0		Q=1	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1
D F/F	D	0	1	0	1
T F/F	T	0	1	1	0
S-R F/F	S	0	1	0	x
	R	x	0	1	0
J-K F/F	J	0	1	x	x
	K	x	x	1	0

Derivation of F/F Input Equations (9/22)

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	×

Q+

J-K F/F Input Map

AB \ Q	0	1
00	0	×
01	1	×
11	0	×
10	1	×

$J = A'B + AB'$

AB \ Q	0	1
00	×	0
01	×	1
11	×	1
10	×	×

$K = B$

Derivation of F/F Input Equations (10/22)

Ex :

P.S.	N.S. x=0	N.S. x=1	Z x=0	Z x=1
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

7 states $\Rightarrow$ 3 F/Fs $\Rightarrow$ A, B, C

Let $S_0 = 000, S_1 = 110,$
$S_2 = 001, S_3 = 111,$
$S_4 = 011, S_5 = 101,$
$S_6 = 010$

Derivation of F/F Input Equations (11/22)

Transition table

			$A^+B^+C^+$		Z	
A	B	C	x = 0	x = 1	x = 0	x = 1
0	0	0	110	001	0	0
1	1	0	111	001	0	0
0	0	1	110	011	0	0
1	1	1	101	001	0	0
0	1	1	110	010	0	0
1	0	1	101	001	1	0
0	1	0	110	010	0	1

Derivation of F/F Input Equations (12/22)

(1) Using D F/F

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
110	1	0	1	0	1	1
001	1	0	1	1	0	1
111	1	0	0	0	1	1
011	1	0	1	1	0	0
101	1	0	0	0	1	1
010	1	0	1	1	0	0

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

Derivation of F/F Input Equations (13/22)

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$$A^+ = D_A = X'$$

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

$$B^+ = D_B$$
$$= X'C' + A'C + A'B$$

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

$$C^+ = D_C = A + XB'$$

Q^+	D
0	0
1	1

Derivation of F/F Input Equations (14/22)

(2) Choose S-R F/F

	$A^+B^+C^+$		S_AR_A		S_BR_B		S_CR_C	
	X=		X=		X=		X=	
ABC	0	1	0	1	0	1	0	1
000	110	001	1 0	0 x	:	:	:	:
110	111	001	x 0	0 1	:	:	:	:
001	110	011	1 0	0 x	:	:	:	:
111	101	001	x 0	0 1	:	:	:	:
011	110	010	1 0	0 x	:	:	:	:
101	101	001	x 0	0 1	:	:	:	:
010	110	010	1 0	0 x	:	:	:	:

Q	Q^+	S	R
0	0	0	×
0	1	1	0
1	0	0	1
1	1	×	0

Derivation of F/F Input Equations (15/22)

(3) Using J-K F/F

ABC	$A^+B^+C^+$ X=0	$A^+B^+C^+$ X=1	J_AK_A X=0	J_AK_A X=1	J_BK_B X=0	J_BK_B X=1	J_CK_C X=0	J_CK_C X=1
000	110	001	1×	0×	1×	0×	0×	1×
110	111	001	×0	×1	×0	×1	1×	1×
001	110	011	1×	1×	:	:	:	:
111	101	001	×0	×1	:	:	:	:
011	110	010	1×	×0	:	:	:	:
101	101	001	×0	0×	:	:	:	:
010	110	010	×0	×0	:	:	:	:

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Derivation of F/F Input Equations (16/22)

Next State Maps

ABC	A+ (X=0)	A+ (X=1)	B+ (X=0)	B+ (X=1)	C+ (X=0)	C+ (X=1)
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

Derivation of F/F Input Equations (17/22)

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

A^+

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$J_A = X'$

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	×	×	0
11	1	×	×	0
10	1	×	×	0

$K_A = X$

BC \ XA	00	01	11	10
00	×	×	×	×
01	×	0	1	×
11	×	0	1	×
10	×	0	1	×

Derivation of F/F Input Equations (18/22)

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

B^+

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	×	×	×	×
10	×	×	×	×

$J_B = X'A' + A'C$

BC \ XA	00	01	11	10
00	×	×	×	×
01	×	×	×	×
11	0	1	1	0
10	0	0	1	0

$K_B = AC + XA$

Derivation of F/F Input Equations (19/22)

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

C^+

BC \ XA	00	01	11	10
00	0	×	×	1
01	×	×	×	×
11	×	×	×	×
10	0	1	1	0

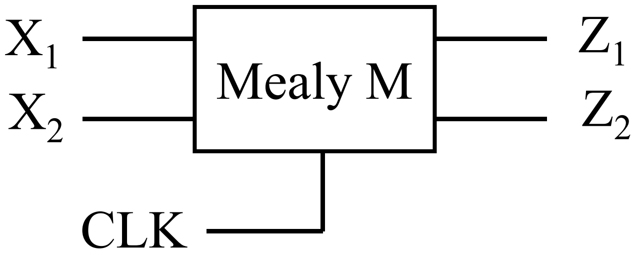
$J_C = A + XB'$

BC \ XA	00	01	11	10
00	×	×	×	×
01	1	0	0	0
11	1	0	0	1
10	×	×	×	×

$K_C = X'A' + A'B$

Derivation of F/F Input Equations (20/22)

Ex: Another example with 2 inputs X_1, X_2, 2 outputs Z_1, Z_2



P.S.	N.S. x_1x_2= 00	01	11	10	Z x_1x_2= 00	01	11	10
S_0	S_0	S_0	S_1	S_1	00	00	01	01
S_1	S_1	S_3	S_2	S_1	00	10	10	00
S_2	S_3	S_3	S_2	S_2	11	11	00	00
S_3	S_0	S_3	S_2	S_0	00	00	00	00

4 states ⇨ 2 F/Fs A, B

State Assignment: $S_0 = 00$, $S_1 = 01$, $S_2 = 11$, $S_3 = 10$

Derivation of F/F Input Equations (21/22)

Transition table

P.S.	N.S. $x_1x_2=$				Z $x_1x_2=$			
AB	00	01	11	10	00	01	11	10
00	00	00	01	01	00	00	01	01
01	01	10	11	01	00	10	10	00
11	10	10	11	11	11	11	00	00
10	00	10	11	00	00	00	00	00

Derivation of F/F Input Equations (22/22)

D F/F

AB \ X_1X_2	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	1	1
10	0	1	1	0

$$D_A = A^+ = X_2B + AB + X_2A$$

AB \ X_1X_2	00	01	11	10
00	0	0	1	1
01	1	0	1	1
11	0	0	1	1
10	0	0	1	0

$$D_B = B^+ = X_1A' + X_2'A'B + X_1B + X_1X_2$$

AB \ X_1X_2	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	0
10	0	0	0	0

$$Z_1 = X_2A'B + X_1'AB$$

AB \ X_1X_2	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	1	0	0
10	0	0	0	0

$$Z_2 = X_1A'B' + X_1'AB$$

Equivalent State Assignments (1/3)

State Graph

⇨ State Table

⇨ State Assignment

⇨ State Transition Table

← (Choice of F/F)

⇨ K-Map

⇨ F/F Input Equations

Different state assignments
⇒ different F/F Input Equations !

Equivalent State Assignments (2/3)

Cost of logic strongly depend on state assignments

Ex:

Assign.1 AB	Assign. 2 AB
00	11
01	10
10	01

P.S.	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
S_1	S_1	S_3	0	0
S_2	S_2	S_1	0	1
S_3	S_2	S_3	1	0

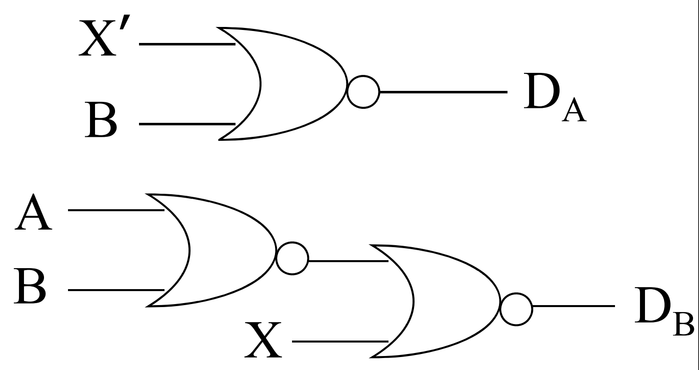
* Assignment 1

$S_1 = 00$, $S_2 = 01$, $S_3 = 10$

D F/F:2 F/Fs A, B

$D_A = XB'$ $D_B = X'(A+B)$

NOR implementa tion



Equivalent State Assignments (3/3)

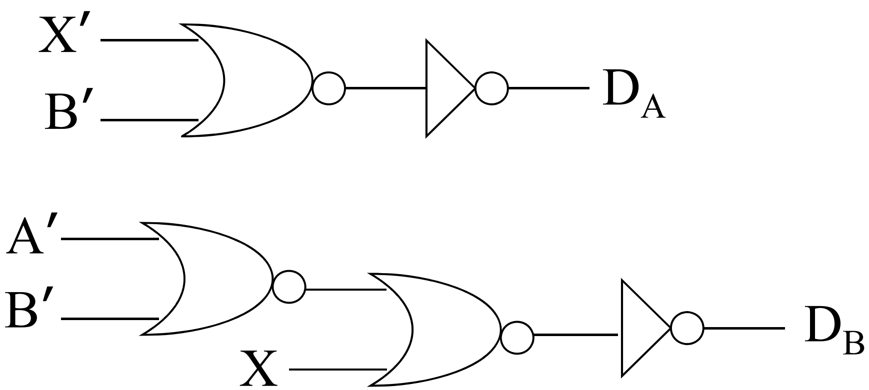
* Assignment 2

$S_1 = 11, S_2 = 10, S_3 = 01$

D F/F:2 F/Fs A, B

$D_A = X' + B' \quad D_B = X + A'B'$

NOR implementa tion



$\Rightarrow$ 2 more gates for NOR implementation

Guidelines for State Assignment (1/16)

Adjacent states: differ in only one variable

2 states:

$S_1(\,0\;1\;0\,)$ $(\,1\;0\;0\,)$	$S_2(\,0\;1\;1\,)$ $(\,1\;1\;0\,)$	adjacent (S_1, S_2)
$S_1(\,0\;1\;0\,)$ $(\,0\;0\;1\,)$	$S_2(\,1\;1\;1\,)$ $(\,1\;1\;1\,)$	not adjacent

Guidelines for State Assignment (2/16)

1. States which have the same next state should be given adjacent assignment
2. States which are next states of the same state should be given adjacent assignment
3. States which have the same output should be given adjacent assignment

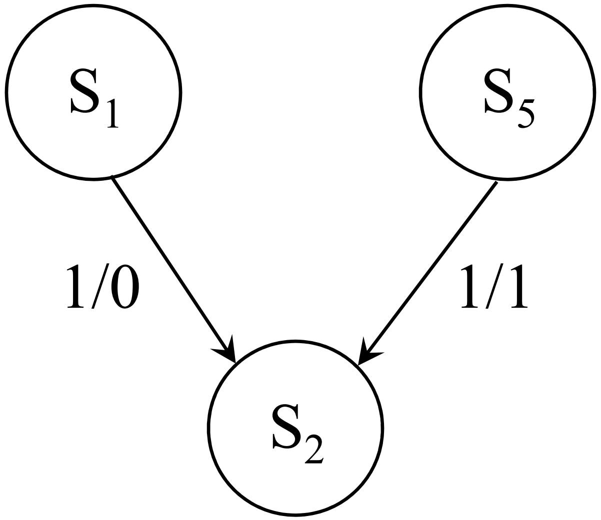
To minimize output function

In above order !!

No guarantee a minimum solution

Guidelines for State Assignment (3/16)

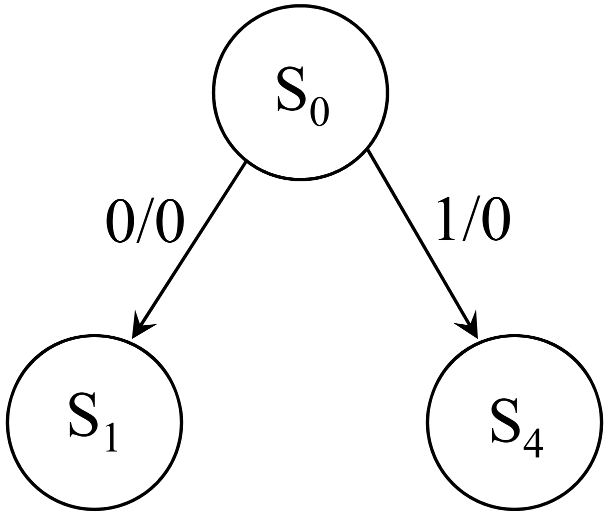
***Guideline 1*：states have the same n.s. with the same input**



S_1, S_5 are adjacent

Guidelines for State Assignment (4/16)

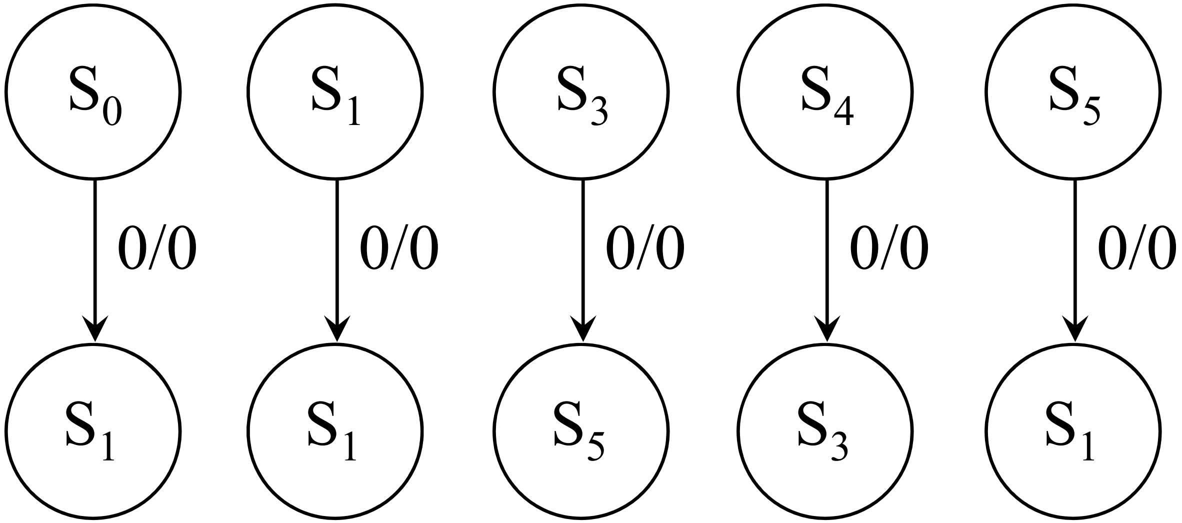
***Guideline 2*：states have the same p.s.**



S_1, S_4 are adjacent

Guidelines for State Assignment (5/16)

***Guideline 3*：states have the same <u>output</u> with the same input**



S_0, S_1, S_3, S_4, S_5 are adjacent

Guidelines for State Assignment (6/16)

1. List relationship

Ex:

3 F/Fs A, B, C

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

States having the same n.s.

Rule 1 : $(S_0, S_2, S_4, S_6)(S_3, S_5)(S_0, S_1, S_3, S_5)(S_4, S_6)$

Rule 2 : $(S_1, S_2)(S_2, S_3)(S_1, S_4)(S_2, S_5)$ x 2 (S_1, S_6) x 2

States having the same p.s.

Guidelines for State Assignment (7/16)

2. Use an assignment map

Assignment #1

BC \ A	0	1
00	S_0	
01	S_2	S_5
11	S_4	S_3
10	S_6	S_1

Assignment #2

BC \ A	0	1
00	S_0	
01	S_1	S_6
11	S_3	S_4
10	S_5	S_2

States having the same n.s.

Rule 1 : $(S_0, S_2, S_4, S_6)(S_3, S_5)(S_0, S_1, S_3, S_5)(S_4, S_6)$

High priority

Rule 2 : $(S_1, S_2)(S_2, S_3)(S_1, S_4)(S_2, S_5)$ x 2 (S_1, S_6) x 2

States having the same p.s.

Guidelines for State Assignment (8/16)

Assignment #1

	P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
000	S_0	S_1	S_2	0	0
110	S_1	S_3	S_2	0	0
001	S_2	S_1	S_4	0	0
111	S_3	S_5	S_2	0	0
011	S_4	S_1	S_6	0	0
101	S_5	S_5	S_2	1	0
010	S_6	S_1	S_6	0	1

6 gates, 13 literals

Guidelines for State Assignment (9/16)

Another straight forward assignment (binary)

	P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
000	S_0	S_1	S_2	0	0
001	S_1	S_3	S_2	0	0
010	S_2	S_1	S_4	0	0
011	S_3	S_5	S_2	0	0
100	S_4	S_1	S_6	0	0
101	S_5	S_5	S_2	1	0
110	S_6	S_1	S_6	0	1

10 gates, 39 literals

Guidelines for State Assignment (10/16)

Why guideline help ?

ABC		$A^+B^+C^+$ x=0		x=1		Z x=0	x=1
000	S_0	110	S_1	001	S_2	0	0
110	S_1	111	S_3	001	S_2	0	0
001	S_2	110	S_1	011	S_4	0	0
111	S_3	101	S_5	001	S_2	0	0
011	S_4	110	S_1	010	S_6	0	0
101	S_5	101	S_5	001	S_0	1	0
010	S_6	110	S_1	010	S_6	0	1

Reordering →

State transition table

		$A^+B^+C^+$ x=0		x=1		Z x=0	x=1
S_0	000	110	S_1	001	S_2	0	*0*
S_2	001	110	S_1	011	S_4	0	*0*
S_6	010	110	S_1	010	S_6	0	*1*
S_4	011	110	S_1	010	S_6	0	*0*
–	100	×××		×××		–	–
S_5	101	*101*	S_5	*001*	S_2	*1*	*0*
S_1	110	*111*	S_3	*001*	S_2	*0*	*0*
S_3	111	*101*	S_5	*001*	S_2	*0*	*0*

Guidelines for State Assignment (11/16)

BC \ XA	00	01	11	11
00	S_1 (S_0)	×	×	S_2 (S_0)
01	S_1 (S_2)	S_5 (S_5)	S_2 (S_5)	S_4 (S_2)
11	S_1 (S_4)	S_5 (S_3)	S_2 (S_3)	S_6 (S_4)
10	S_1 (S_6)	S_3 (S_1)	S_2 (S_1)	S_6 (S_6)

$S_0S_2S_4S_6$ S_3S_5 $S_1S_3S_5$ S_4S_6

A^+

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$A^+ = X'$

B^+

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

C^+

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

due to S_2 S_5 assignment

Guidelines for State Assignment (12/16)

Ex:

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
a	a	c	0	0
b	d	f	0	1
c	c	a	0	0
d	d	b	0	1
e	b	f	1	0
f	c	e	1	0

Guidelines

1. *(b,d)(c,f)(b,e)*
2. *(a,c)* x 2 , *(d,f)(b,d)*
   (b,f)(c,e)
3. *(a,c)(b,d)(e,f)*

Guidelines for State Assignment (13/16)

Assignment #1

Q_2Q_3 \ Q_1	0	1
00	a	c
01		e
11	b	d
10		f

a = 000
b = 011
c = 100
d = 111
e = 101
f = 110

1. $(b,d)(c,f)(b,e)$
2. $(a,c) \times 2$, $(d,f)(b,d)$
   $(b,f)(c,e)$
3. $(a,c)(b,d)(e,f)$

Assignment #2

Q_2Q_3 \ Q_1	0	1
00	c	a
01		e
11	d	b
10	f	

a = 100
b = 111
c = 000
d = 011
e = 101
f = 010

1. $(b,d)(c,f)(b,e)$
2. $(a,c) \times 2$, $(d,f)(b,d)$
   $(b,f)(c,e)$
3. $(a,c)(b,d)(e,f)$

For assignment #2

	P.S.	N.S.		Z	
	$Q_1Q_2Q_3$	$x=0$	$x=1$	$x=0$	$x=1$
m_4	100	100	000	0	0
m_7	111	011	010	0	1
m_0	000	000	100	0	0
m_3	011	011	111	0	1
m_5	101	111	010	1	0
m_2	010	000	101	1	0
m_1	001	– – –	– – –	–	–
m_6	110	– – –	– – –	–	–

Guidelines for State Assignment (15/16)

State transition table

	P.S. $Q_1Q_2Q_3$	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
m_0	000	000	100	0	0
m_1	001	---	---	–	–
m_2	010	000	101	1	0
m_3	011	011	111	0	1
m_4	100	100	000	0	0
m_5	101	111	010	1	0
m_6	110	---	---	–	–
m_7	111	011	010	0	1

next state map

Q_2Q_3 \ XQ_1	00	01	11	10
00	000	100	000	100
01	×	111	010	×
11	011	011	010	111
10	000	×	×	101

D F/F Implementation

Q_2Q_3 \ XQ_1	00	01	11	10
00		1		1
01	×	1		×
11				1
10		×	×	1

$$Q_1^+ = X'Q_1Q_2' + XQ_1'$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1	1	×
11	1	1	1	1
10		×	×	

$$Q_2^+ = Q_3$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1		×
11	1	1		1
10		×	×	1

$$Q_3^+ = X'Q_3 + XQ_1'Q_2$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1		×
11			1	1
10	1	×	×	

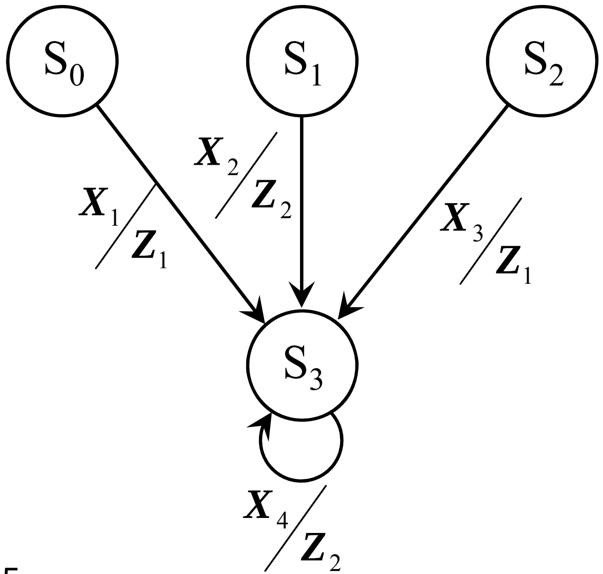
$$Z = X'Q_2'Q_3 + X'Q_2Q_3' + XQ_2Q_3$$

Using a One-Hot State Assignment (1/2)

- Design with FPGAs, not to minimize the number of F/F
- Reduce the number of logic cells and interconnections between cells to minimize delay

One-hot state assignment

- Using one F/F only for each state
- 4 states use 4 F/Fs (Q_0, Q_1 , Q_2 , Q_3) with following state assignment
  S_0: $Q_0Q_1Q_2Q_3$=1000, **S_1**: 0100, **S_2**: 0010, **S_3**: 0001



Write next-state and output eq. directly by inspecting the state graph

$$Q_3^+ = X_1Q_0 + X_2Q_1 + X_3Q_2 + X_4Q_3$$

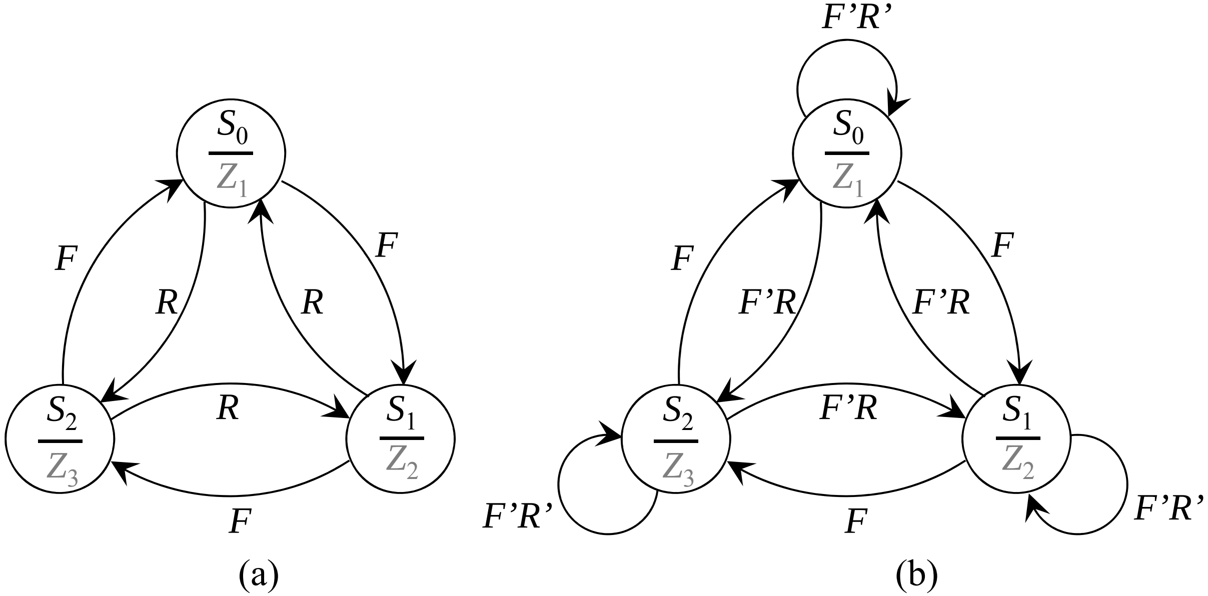
$$Z_1 = X_1Q_0 + X_3Q_2$$

$$Z_2 = X_2Q_1 + X_4Q_3$$

Using a One-Hot State Assignment (2/2)

- For Moore machine,

S_0: $Q_0Q_1Q_2$=100, **S_1**: 010, **S_2**: 001



$$Q_0^+ = F'R'Q_0 + FQ_2 + F'RQ_1$$

$$Q_1^+ = F'R'Q_1 + FQ_0 + F'RQ_2$$

$$Q_2^+ = F'R'Q_2 + FQ_1 + F'RQ_0$$

$Z_1 = Q_0 \quad Z_2 = Q_1 \quad Z_3 = Q_2$